Project GRAD Mathematics
Research Foundations I

What we do:

Provide all students an opportunity to learn mathematics through provision of adequate instructional time and identification and selection of rigorous instructional activities for teachers to utilize on a daily basis

Why we do it:

The extent of the students' opportunity to learn mathematics content bears directly and decisively on student mathematics achievement.

What the research says:

The term 'opportunity to learn' (OTL) refers to what is studied or embodied in the tasks that students perform. In mathematics, OTL includes the scope of the mathematics presented, how the mathematics is taught, and the match between students' entry skills and new material. The strong relationship between OTL and student performance in mathematics has been documented in many research studies. The concept was studied in the First International Mathematics Study (Husén), where teachers were asked to rate the extent of student exposure to particular mathematical concepts and skills.

Strong correlations were found between student OTL scores and mean student achievement scores in mathematics, with high OTL scores associated with high achievement. The link between student mathematics achievement and opportunity to learn was also found in subsequent international studies, such as the Second International Mathematics Study (McKnight et al.) and the Third International Mathematics and Science Study (TIMSS) (Schmidt, McKnight & Raizen). As might be expected, there is also a positive relationship between total time allocated to mathematics and general mathematics achievement. Suarez et al., in a review of research on instructional time, found strong support for the link between allocated instructional time and student performance. Internationally, Keeves found a significant relationship across Australian states between achievement in mathematics and total curriculum time spent on mathematics.

In spite of these research findings, many students still spend only minimal amounts of time in the mathematics class. For instance, Grouws and Smith, in an analysis of data from the 1996 National Assessment of Educational Progress (NAEP) mathematics study, found that 20% of eighth-grade students had thirty minutes or less for mathematics instruction each day.

Adapted from *Improving Student Achievement in Mathematics* by Douglas A. Grouws and Kristin J. Cebulla, International Academy of Education.

Research has also found a strong relationship between mathematics-course taking at the secondary school level and student achievement. Reports from the NAEP in mathematics showed that 'the number of advanced mathematics courses taken was the most powerful predictor of students' mathematics performance after adjusting for variations in home background'. Textbooks are also related to student OTL, because many textbooks do not contain much content that is new to students. The lack of attention to new material and heavy emphasis on review in many textbooks are of particular concern at the elementary school and middle-school levels. Flanders examined several textbook series and found that fewer than 50% of the pages in textbooks for grades two through eight contained any material new to students. In a review of a dozen middle-grade mathematics textbook series, Kulm, Morris and Grier found that most traditional textbook series lack many of the content recommendations made in recent standards documents.

United States data from TIMSS showed important differences in the content taught to students in different mathematics classes or tracks. For example, students in remedial classes, typical eighth-grade classes and pre-algebra classes were exposed to very different mathematics contents, and their achievement levels varied accordingly. The achievement tests used in international studies and in NAEP assessments measure important mathematical outcomes and have commonly provided a broad and representative coverage of mathematics. Moreover, the tests have generally served to measure what even the most able students know and do not know. Consequently, they provide reasonable outcome measures for research that examines the importance of opportunity to learn as a factor in student mathematics achievement.

Implications in the classroom

The findings about the relationship between opportunity to learn and student achievement have important implications for teachers. In particular, it seems prudent to allocate efficient time for mathematics instruction at every grade level. Short class periods in mathematics, instituted for whatever practical or philosophical reason, should be seriously questioned. Of special concern are the 30–35 minute class periods for mathematics being implemented in some middle schools.

Textbooks that devote major attention to review and that address little new content each year should be avoided, or their use should be heavily supplemented in appropriate ways. Teachers should use textbooks as just one instructional tool among many, rather than feel duty-bound to go through the textbook on a one-section-per-day basis.

Teachers must ensure that students are given the opportunity to learn important content and skills. If students are to compete effectively in a global, technologically oriented society, they must be taught the mathematical skills needed to do so. Thus, if problem solving is essential, explicit attention must be given to it on a regular and sustained basis.

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If we expect students to develop number sense, it is important to attend to mental computation and estimation as part of the curriculum. If proportional reasoning and deductive reasoning are important, attention must be given to them in the curriculum implemented in the classroom.

It is important to note that opportunity to learn is related to equity issues. Some educational practices differentially affect particular groups of students' opportunity to learn. For example, a recent American Association of University Women study showed that boys' and girls' use of technology is markedly different. Girls take fewer computer science and computer design courses than do boys. Furthermore, boys often use computers to program and solve problems, whereas girls tend to use the computer primarily as a word processor. This suggests that, as technology is used in the mathematics classroom, teachers must assign tasks and responsibilities to students in such a way that both boys and girls have active learning experiences with the technological tools employed.

OTL is also affected when low-achieving students are tracked into special 'basic skills' curricula, oriented towards developing procedural skills, with little opportunity to develop problem solving and higher-order thinking abilities. The impoverished curriculum frequently provided to these students is an especially serious problem because the ideas and concepts frequently untaught or de-emphasized are the very ones needed in every day life and in the workplace.

References

American Association of University Women, 1998; Atanda, 1999; Flanders, 1987; Grouws & Smith, in press; Hawkins, Stancavage & Dossey, 1998; Husén, 1967; Keeves, 1976, 1994; Kulm, Morris & Grier, 1999; McKnight et al., 1987; Mullis, Jenkins & Johnson, 1994; National Center for Education Statistics, 1996, 1997, 1998; Schmidt, McKnight & Raizen, 1997; Secada, 1992; Suarez et al., 1991.

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Project GRAD Mathematics
Research Foundations II

What we do:

Emphasize teaching mathematics for meaning and understanding

Why we do it:

Focusing instruction on the meaningful development of important mathematical ideas increases the level of student learning.

What the research says:

There is a long history of research, going back to the 1940s and the work of William Brownell, on the effects of teaching for meaning and understanding in mathematics. Investigations have consistently shown that an emphasis on teaching for meaning has positive effects on student learning, including better initial learning, greater retention and an increased likelihood that the ideas will be used in new situations. These results have also been found in studies conducted in high-poverty areas.

Implications in the classroom

As might be expected, the concept of 'teaching for meaning' has varied somewhat from study to study, and has evolved over time. Teachers will want to consider how various interpretations of this concept can be incorporated into their classroom practice.

• *Emphasize the mathematical meanings of ideas, including how the idea, concept or skill is connected in multiple ways to other mathematical ideas in a logically consistent and sensible manner.* Thus, for subtraction, emphasize the inverse, or 'undoing', relationship between it and addition. In general, emphasis on meaning was common in early research in this area in the late 1930s, and its purpose was to avoid the mathematical meaningfulness of the ideas taught receiving only minor attention compared to a heavy emphasis on the social uses and utility of mathematics in everyday life.

• *Create a classroom learning context in which students can construct meaning.* Students can learn important mathematics both in contexts that are closely connected to real life situations and in those that are purely mathematical. The abstractness of a learning environment and how students relate to it must be carefully regulated, closely monitored and thoughtfully chosen. Consideration should be given to students' interests and backgrounds. The mathematics taught and learned must seem reasonable to students and

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make sense to them. An important factor in teaching for meaning is connecting the new ideas and skills to students' past knowledge and experience.

• *Make explicit the connections between mathematics and other subjects.* For example, instruction could relate data gathering and data-representation skills to public opinion polling in social studies. Or, it could relate the mathematical concept of direct variation to the concept of force in physics to help establish a real-world referent for the idea.

• *Attend to student meanings and student understanding in instruction.* Students' conceptions of the same idea will vary, as will their methods of solving problems and carrying out procedures. Teachers should build on students' intuitive notions and methods in designing and implementing instruction.

References

Aubrey, 1997; Brownell, 1945, 1947; Carpenter et al., 1998; Cobb et al., 1991; Fuson, 1992; Good, Grouws & Ebmeier, 1983; Hiebert & Carpenter,1992;Hiebert & Wearne,1996;Hiebert et al.,1997; Kamii, 1985, 1989, 1994; Knapp, Shields & Turnbull, 1995; Koehler & Grouws, 1992; Skemp, 1978;Van Engen,1949;Wood & Sellers,1996,1997.

Adapted from *Improving Student Achievement in Mathematics* by Douglas A. Grouws and Kristin J. Cebulla, International Academy of Education.

Project GRAD Mathematics
Research Foundations III

What we do:

Emphasize a balanced approach to instruction that encourages both students' conceptual understanding and mastery of skills

Why we do it:

When students understand mathematics, they are able to use their knowledge flexibly. They combine factual knowledge, procedural facility, and conceptual understanding in powerful ways.

What the research says:

Research suggests that students who develop conceptual understanding early perform best on procedural knowledge later. Students with good conceptual understanding are able to perform successfully on near-transfer tasks and to develop procedures and skills they have not been taught. Students without conceptual understanding are able to acquire procedural knowledge when the skill is taught, but research suggests that students with low levels of conceptual understanding need more practice in order to acquire procedural knowledge. Research by Heid suggests that students are able to understand concepts without prior or concurrent skill development. In her research with calculus students, instruction was focused almost entirely on conceptual understanding. Skills were taught briefly at the end of the course. On procedural skills, the students in the conceptual-understanding approach performed as well as those taught with a traditional approach. Furthermore, these students significantly outperformed traditional students on conceptual understanding. Mack demonstrated that students' rote (and frequently faulty) knowledge often interferes with their informal (and usually correct) knowledge about fractions. She successfully used students' informal knowledge to help them understand symbols for fractions and develop algorithms for operations. Fawcett's research with geometry students suggests that students can learn basic concepts, skills and the structure of geometry through problem solving.

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Implications in the classroom

There is evidence that students can learn new skills and concepts while they are working out solutions to problems. For example, armed with only a knowledge of basic addition, students can extend their learning by developing informal algorithms for addition of larger numbers. Similarly, by solving carefully chosen non-routine problems, students can develop an understanding of many important mathematical ideas, such as prime numbers and perimeter/area relations. Development of more sophisticated mathematical skills can also be approached by treating their development as a problem for students to solve. Teachers can use students' informal and intuitive knowledge in other areas to develop other useful procedures. Instruction can begin with an example for which students intuitively know the answer. From there, students are allowed to explore and develop their own algorithm. For instance, most students understand that starting with four pizzas and then eating a half of one pizza will leave three and a half pizzas. Teachers can use this knowledge to help students develop an understanding of subtraction of fractions. Research suggests that it is not necessary for teachers to focus first on skill development and then move on to problem solving. Both can be done together. Skills can be developed on an as-needed basis, or their development can be supplemented through the use of technology. In fact, there is evidence that if students are initially drilled too much on isolated skills, they have a harder time making sense of them later.

References

Cognition and Technology Group, 1997; Fawcett, 1938; Heid, 1988; Hiebert & Wearne, 1996; Mack, 1990; Resnick & Omanson, 1987; Wearne & Hiebert, 1988.

Adapted from *Improving Student Achievement in Mathematics* by Douglas A. Grouws and Kristin J. Cebulla, International Academy of Education.

Project GRAD Mathematics
Research Foundations IV

What we do:

Encourage teachers to allow students to develop unique solutions to mathematics problems, reflect openly about their solutions, interact with other students at high levels in the mathematics classroom

Why we do it:

Teaching that incorporates students' intuitive solution methods can increase student learning, especially when combined with opportunities for student interaction and discussion.

What the research says:

Recent results from the TIMSS video study have shown that Japanese classrooms use student solution methods extensively during instruction. Interestingly, the same teaching technique appears in many successful U.S. research projects. Findings from U.S. studies clearly demonstrate two important principles that are associated with the development of students' deep conceptual understanding of mathematics. First, student achievement and understanding are significantly improved when teachers are aware of how students construct knowledge, are familiar with the intuitive solution methods that students use when they solve problems, and utilize this knowledge when planning and conducting instruction in mathematics. These results have been clearly demonstrated in the primary grades and are beginning to be shown at higher-grade levels. Second, structuring instruction around carefully chosen problems, allowing students to interact when solving these problems, and then providing opportunities for them to share their solution methods result in increased achievement on problem - solving measures. Importantly, these gains come without a loss of achievement in the skills and concepts measured on standardized achievement tests.

Research has also demonstrated that when students have opportunities to develop their own solution methods, they are better able to apply mathematical knowledge in new problem situations.

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Implications in the classroom

Research results suggest that teachers should concentrate on providing opportunities for students to interact in problem-rich situations. Besides providing appropriate problem-rich situations, teachers must encourage students to find their own solution methods and give them opportunities to share and compare their solution methods and answers. One way to organize such instruction is to have students work in small groups initially and then share ideas and solutions in a whole-class discussion. One useful teaching technique is for teachers to assign an interesting problem for students to solve and then move about the room as they work, keeping track of which students are using which strategies (taking notes if necessary). In a wholeclass setting, the teacher can then call on students to discuss their solution methods in a pre-determined and carefully considered order, these methods often ranging from the most basic to more formal or sophisticated ones. This teaching structure is used successfully in many Japanese mathematics lessons.

References

Boaler, 1998; Carpenter et al., 1988, 1989, 1998; Cobb, Yackel & Wood, 1992; Cobb et al., 1991; Cognition and Technology Group, 1997; Fennema, Carpenter & Peterson, 1989; Fennema et al., 1993, 1996; Hiebert & Wearne, 1993, 1996; Kamii, 1985, 1989, 1994; Stigler & Hiebert, 1997; Stigler et al., 1999; Wood, Cobb & Yackel, 1995; Wood et al., 1993;Yackel,Cobb & Wood, 1991.

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Project GRAD Mathematics
Research Foundations V

What we do:

Provide teachers structured opportunities to facilitate small group learning experiences for students on a daily basis

Why we do it:

Using small groups of students to work on activities, problems and assignments can increase student mathematics achievement.

What the research says:

Considerable research evidence within mathematics education indicates that using small groups of various types for different classroom tasks has positive effects on student learning. Davidson, for example, reviewed almost eighty studies in mathematics that compared student achievement in small-group settings with traditional whole-class instruction. In more than 40% of these studies, students in the classes using small-group approaches significantly outscored control students on measures of student performance. In only two of the seventy-nine studies did control-group students perform better than the small-group students, and in these studies there were some design irregularities. From a review of ninety-nine studies of co-operative group-learning methods at the elementary and secondary school levels, Slavin concluded that co-operative methods were effective in improving student achievement. The most effective methods emphasized both group goals and individual accountability. From a review by Webb of studies examining peer interaction and achievement in small groups (seventeen studies, grades 2–11), several consistent findings emerged. First, giving an explanation of an idea, method or solution to a team mate in a group situation was positively related to achievement. Second, receiving 'non-responsive' feedback (no feedback or feedback that is not pertinent to what one has said or done) from team mates was negatively related to achievement. Webb's review also showed that group work was most effective when students were taught how to work in groups and how to give and receive help. Received help was most effective when it was in the form of elaborated explanations (not just the answer) and then applied by the student either to the current problem or to a new problem.

Qualitative investigations have shown that other important and often unmeasured outcomes beyond improved general achievement can result from small-group work.

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In one such investigation, Yackel, Cobb and Wood studied a second-grade classroom in which small-group problem solving followed by whole-class discussion was the primary instructional strategy for the entire school year. They found that this approach created many learning opportunities that do not typically occur in traditional classrooms, including opportunities for collaborative dialogue and resolution of conflicting points of view. Slavin's research showed positive effects of small-group work on cross-ethnic relations and student attitudes towards school.

Implications in the classroom

Research findings clearly support the use of small groups as part of mathematics instruction. This approach can result in increased student learning as measured by traditional achievement measures, as well as in other important outcomes.
When using small groups for mathematics instruction, teachers should:
• choose tasks that deal with important mathematical concepts and ideas;
• select tasks that are appropriate for group work;
• consider having students initially work individually on a task and then follow this with group work where students share and build on their individual ideas and work;
• give clear instructions to the groups and set clear expectations for each;
• emphasize both group goals and individual accountability;
• choose tasks that students find interesting;
• ensure that there is closure to the group work, where key ideas and methods are brought to the surface either by the teacher or the students, or both. Finally, as several research studies have shown, teachers should not think of small groups as something that must always be used or never be used. Rather, small-group instruction should be thought of as an instructional practice that is appropriate for certain learning objectives, and as a practice that can work well with other organizational arrangements, including whole-class instruction.

References

Cohen, 1994; Davidson, 1985; Laborde, 1994; Slavin, 1990, 1995; Webb, 1991; Webb, Troper & Fall, 1995;Yackel,Cobb & Wood, 1991.

Adapted from *Improving Student Achievement in Mathematics* by Douglas A. Grouws and Kristin J. Cebulla, International Academy of Education.

Project GRAD Mathematics
Research Foundations VI

What we do:

Encourage appropriate student use of manipulatives and tools

Why we do it:

Long-term, appropriate use of concrete materials is positively related to increases in student mathematics achievement and improved attitudes towards mathematics.

What the research says:

Many studies show that the use of concrete materials can produce meaningful use of notational systems and increase student concept development. In a comprehensive review of activity based learning in mathematics in kindergarten through grade eight, Suydam and Higgins concluded that using manipulative materials produces greater achievement gains than not using them. In a more recent meta-analysis of sixty studies (kindergarten through post-secondary) that compared the effects of using concrete materials with the effects of more abstract instruction, Sowell concluded that the long-term use of concrete instructional materials by teachers knowledgeable in their use improved student achievement and attitudes. In spite of generally positive results, there are some inconsistencies in the research findings. As Thompson points out, the research results concerning concrete materials vary, even among treatments that were closely controlled and monitored and that involved the same concrete materials. For example, in studies by Resnick and Omanson and by Labinowicz, the use of base-ten blocks showed little impact on children's learning. In contrast, both Fuson and Briars and Hiebert and Wearne reported positive results from the use of base-ten blocks. The differences in results among these studies might be due to the nature of the students' engagement with the concrete materials and their orientation towards the materials in relation to notation and numerical values. They might also be due to different orientations in the studies, with regard to the role of computational algorithms and how they should be developed in the classroom. In general, however, the ambiguities in some of the research findings do not undermine the general consensus that concrete materials are valuable instructional tools.

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Implications in the classroom

Although successful teaching requires teachers to carefully choose their procedures on the basis of the context in which they will be used, available research suggests that teachers should use manipulative materials in mathematics instruction more regularly in order to give students hands-on experience that helps them construct useful meanings for the mathematical ideas they are learning. Use of the same material to teach multiple ideas over the course of schooling has the advantage of shortening the amount of time it takes to introduce the material and also helps students to see connections between ideas.

The use of concrete material should not be limited to demonstrations. It is essential that children use materials in meaningful ways rather than in a rigid and prescribed way that focuses on remembering rather than on thinking. Thus, as Thompson says, 'before students can make productive use of concrete materials, they must first be committed to making sense of their activities and be committed to expressing their sense in meaningful ways. Further, it is important that students come to see the two-way relationship between concrete embodiments of a mathematical concept and the notational system used to represent it.'

References

Fuson & Briars, 1990; Hiebert & Wearne, 1992; Labinowicz, 1985; Leinenbach & Raymond, 1996; Resnick & Omanson, 1987; Sowell, 1989; Suydam & Higgins, 1977; Thompson, 1992; Varelas & Becker, 1997. 28

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